



A ball is dropped off the CN Tower and its velocity is modeled by the function $h(t) = -4.9t^2 + 600$, where h is the height of the ball in metres, and t is the time in seconds after the ball's release.

a) Calculate the average velocity of the ball over the first four seconds of flight.

$$t: 0 \text{ to } t=4$$

b) Calculate the velocity of the ball at exactly 4.0 seconds into its decent.

$$h(4) = -4.9(4)^2 + 600 \\ = 521.6$$

$$\begin{aligned} \text{a) AROC} &= \frac{h(t_1) - h(t_2)}{t_1 - t_2} \\ &= \frac{h(0) - h(4)}{0 - 4} \\ &= \frac{600 - (521.6)}{-4} \end{aligned}$$

$$= \frac{78.4}{-4}$$

$$= -19.6 \text{ m/s}$$

$$\text{IROC} = \frac{h(t_1) - h(t_2)}{t_1 - t_2}$$

$$\text{IROC} = \frac{h(3.99) - h(4)}{3.99 - 4}$$

$$\text{IROC} = \frac{521.99151 - 521.6}{-0.01}$$

$$\text{IROC} = \frac{0.39151}{-0.01}$$

$$= -39.151 \text{ m/s}$$

$$\begin{aligned} h(3.99) &= -4.9(3.99)^2 + 600 \\ &= 521.99151 \end{aligned}$$

$$-9.8(4)$$

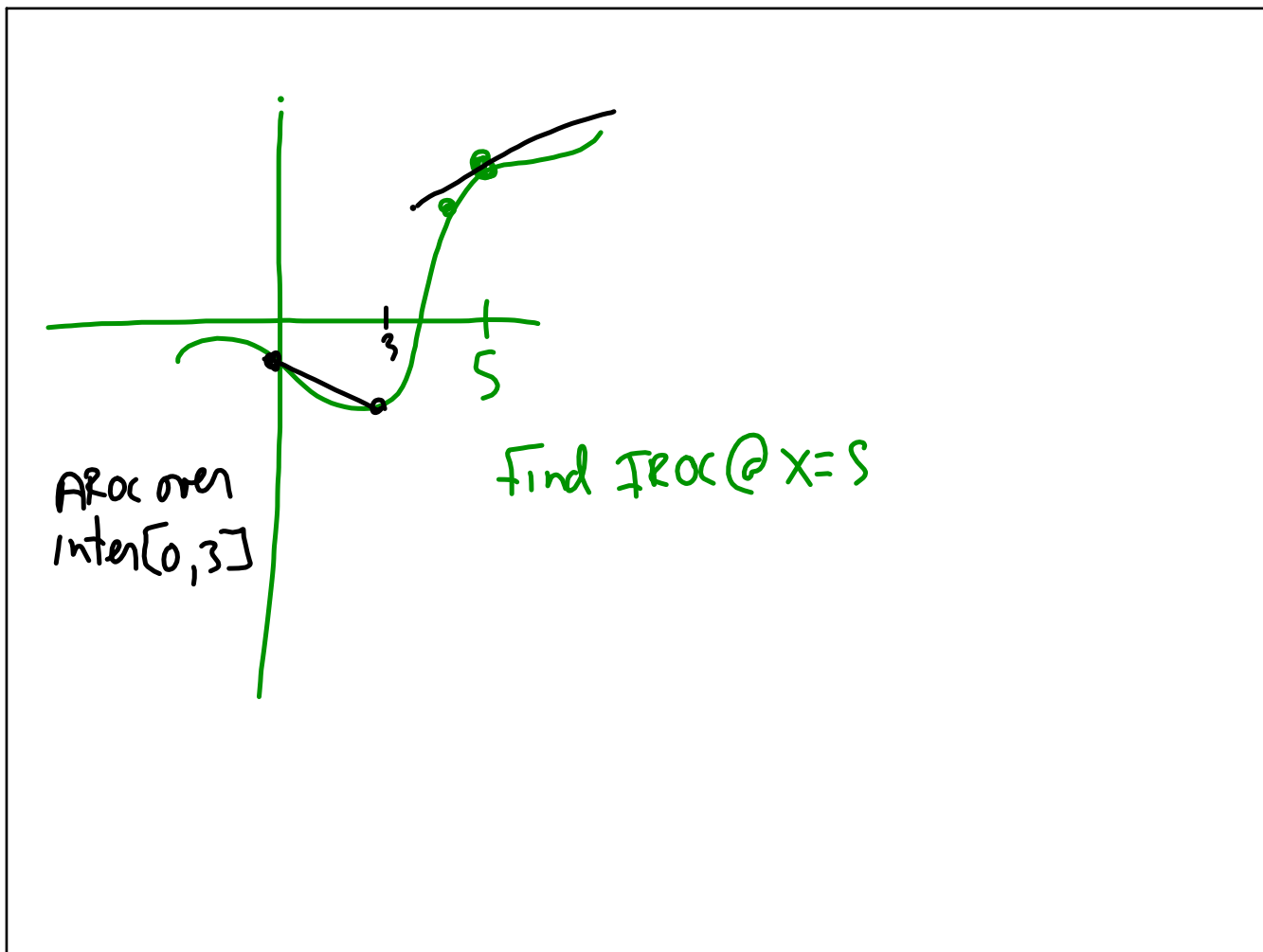
$$= -39.2$$

Calculus 120

Unit 1: Rate of Change and Derivatives

February 1, 2019: Day #3

- 1. Quick quiz Tuesday - AROC and IROC**
- 2. Signed Course Outlines**
- 3. Assignment Due Monday**
- 4. Review Questions Page 66-67 #3, 4, 69b, 70c**



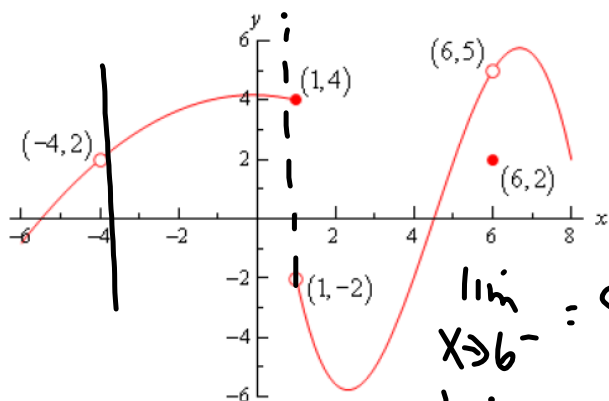
Curriculum Outcomes

C1. Explore the concepts of average and instantaneous rate of change.

LIMIT REVIEW

A function has a limit as x approaches c if the closer you get to c from each side, you approach the same value of the function. Limits can also be taken only from the left or right as well in certain situations, for example the endpoint of a radical function.

Q: Given the graph of $f(x)$, find the following limits:



$$\left. \begin{array}{l} \lim_{x \rightarrow 6^-} f(x) = 5 \\ \lim_{x \rightarrow 6^+} f(x) = 5 \end{array} \right\} \lim_{x \rightarrow 6} f(x) = 5$$

- a) $\lim_{x \rightarrow -4^-} f(x) = 2$
- b) $\lim_{x \rightarrow -4^+} f(x) = 2$
- c) $\lim_{x \rightarrow -4} f(x) = 2$
- d) $\lim_{x \rightarrow 1^-} f(x) = 4$
- e) $\lim_{x \rightarrow 1^+} f(x) = -2$
- f) $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

We can calculate limits algebraically by a number of means.

- 1. Direct substitution of c .**
- 2. Factoring to eliminate a zero denominator.**
- 3. Rationalizing the numerator or denominator (multiplying by a conjugate)**
- 4. Finding common denominators.**
- 5. Trial and error if all else fails (substitute numbers closer and closer to c from both sides).**

$$\lim_{x \rightarrow 5} x^2 + 2x - 3$$

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$$

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 3x + 2}$$

$$\frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}(x-1)}$$

$$\lim_{x \rightarrow 0} \frac{(2+h)^2 - 4}{h}$$

$$\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3} =$$

$$\frac{(\cancel{\sqrt{x}-3})(\sqrt{x}+3)}{\cancel{\sqrt{x}-3}}$$

$$\frac{4}{\sqrt{x+2}-3} \cdot \frac{\sqrt{x+2}+3}{\sqrt{x+2}+3}$$

$$\frac{4(\sqrt{x+2}+3)}{x+2-9}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{1+x} - \frac{1}{1}}{x}$$

$$\lim_{x \rightarrow 0} \frac{1}{1+x} - \frac{(1+x)}{1+x}$$

$$\lim_{x \rightarrow 0} \frac{1 - (1+x)}{1+x}$$

$$\lim_{x \rightarrow 0} \frac{-x}{1+x}$$

$$\lim_{x \rightarrow 0} \left(\frac{-x}{1+x} \right) \left(\frac{1}{x} \right)$$

$$\lim_{x \rightarrow 0} \frac{-1}{1+x}$$

$$= -1$$

$$\frac{3}{7} + \frac{2}{7}$$

$$\frac{3+2}{7}$$

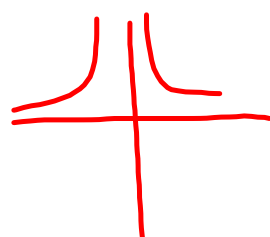
$$\lim_{x \rightarrow 0} \frac{1}{x^2}$$

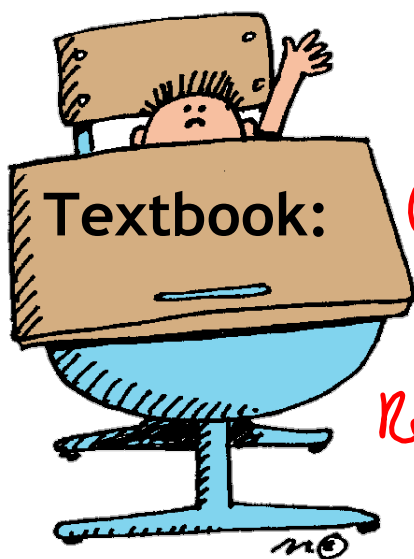
Left

X	Y
-1	1
-0.5	4
-0.1	100
-0.01	10000
-0.0001	⋮
	∞

Right

X	Y
1	1
0.5	4
0.1	100
0.01	⋮
	∞

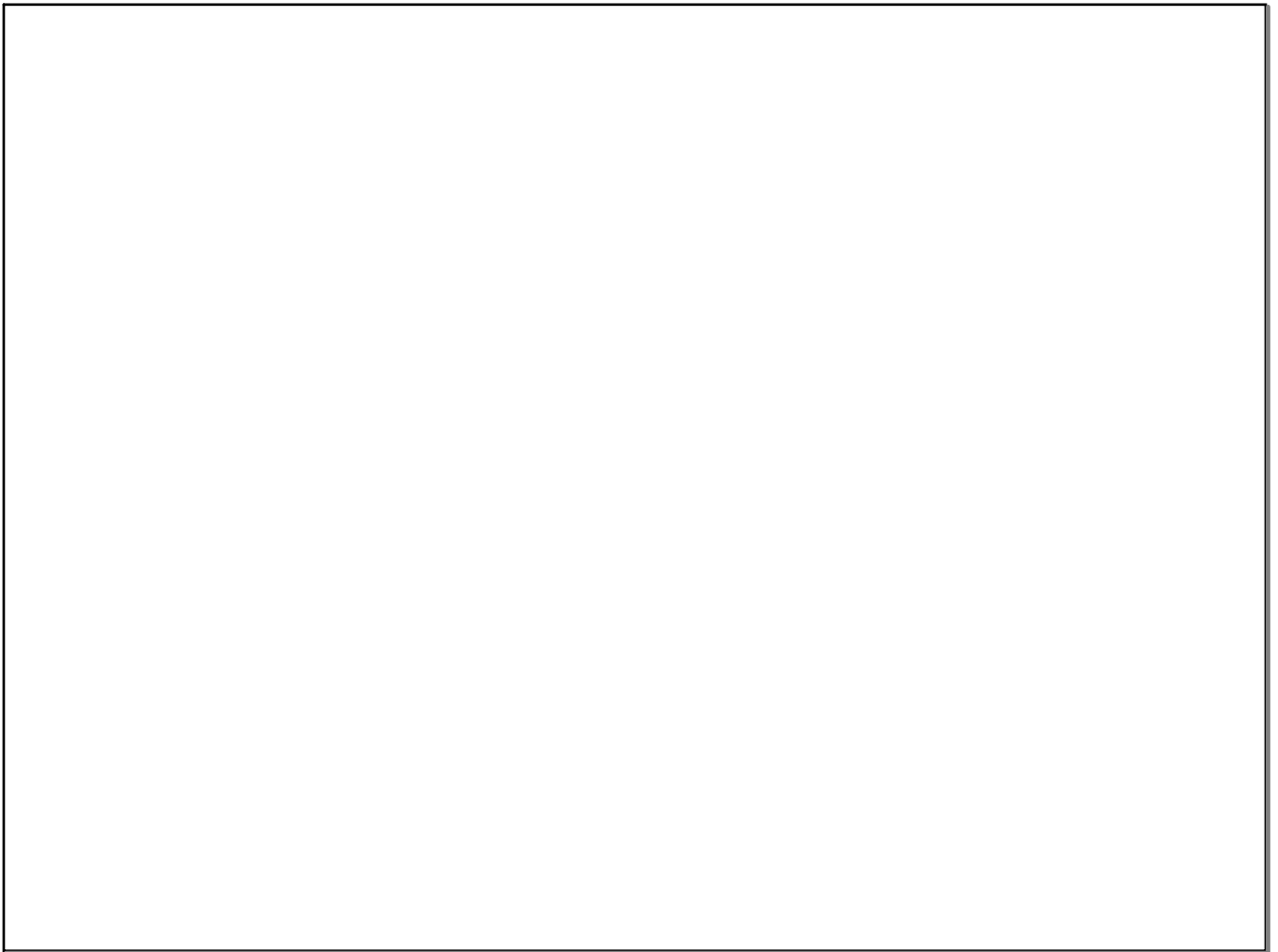




Practice

Page 66-67 #7, 13, 18
{25, 27, 29 algebra},
31 (trial and error table),
43, 45, 47,

Extra Practice: Red Book
p. 19 # 4 aceg, 5 ace,
6 ace,



Ex: Find the AROC of the function
 $y = 3x^2 - 2x + 1$ from $x = 6$ to $x = 8$

$$\begin{aligned} \text{AROC} &= \frac{f(x_1) - f(x_2)}{x_1 - x_2} \\ &= \frac{f(6) - f(8)}{6 - 8} \\ &= \frac{97 - 177}{-2} \\ &= \frac{-80}{-2} \\ &= 40 \end{aligned}$$

$$\begin{aligned} f(6) &= 3(6)^2 - 2(6) + 1 \\ &= 97 \\ f(8) &= 3(8)^2 - 2(8) + 1 \\ &= 177 \end{aligned}$$

Attachments

2.1_74_AP.html



2.1_74_AP.swf



2.1_74_AP.html